

The language of Equations

Word	Meaning	Example
Equation	An algebraic expression which contains an equal (=) sign	$2x - y = 4(x - b), 5 = 2x + 1$
Linear Equation (with one variable)	An equation that can be written in the form : $ax + b = 0$ ($a \neq 0$)	$3x+5 = 0$ (this is in classic form) Or $15(x+1)=3$ $15x+15=3$ $15x+15-3=0$ $15x+12=0$ (now this is in classic form)
Solution Set	The set of all numbers that satisfy the equation	$2x - 2 = 0$ <i>Solution set</i> {1}
Identity	An equation that has infinite solutions. (An equations that can be satisfied by all numbers)	$2x - 4 = 5(x - 1) - 3x + 1$
Conditional Equation	An equation that has at least one real number solution and is not an identity	$2x - 6 = 4$
Inconsistent (or Contradiction)	An equation that has no solutions (The solution set is \emptyset)	$2x - 4 = 5(x - 1) - 3x + 2$

How to solve a linear equation?

Perform **inverse operations** on both sides of the equation (to undo how the equation is built up) so that you can **isolate the unknown**

(The inverse of "+" is "-" and the inverse of "" is "÷")*

After you solve it check your solution by substitution

Example

$$2x - 5 = 24$$

(We will try to isolate x on the left side)

$$2x = 24 + 5$$

(To move -5 from the left side to the right we inverted the operation)

$$2x = 29$$

$$\frac{2x}{2} = \frac{29}{2}$$

$$x = \frac{29}{2}$$

(We inverted the operation)

Exercise 1 → Let's practice 😊

Solve for x :

1. $x + 9 = 5$

2. $5x = 40$

3. $15 + 3x = 22$

4. $-3x = 15$

5. $-2x - 2 = 15$

6. $8 = 23 - 5x$

7. $2x - 4 = 5(x - 1) - 3x + 1$

8. $2x - 4 = 5(x - 1) - 3x + 2$

Exercise 2 → Let's practice some more... ;-)

Solve for x:

1. $2x - 3 = 3x + 9$

2. $-2x - 12 = -2x + 4$

3. $x + 2x + 5 = 5 + 3x$

4. $12 - 7x = 3x - 8$

5. $5x - 9 = 7 - 2x$

6. $2(2x + 3) - 3x = 5 + x$

7. $x - (5x + 1) = 2(x - 4) + (-3x)$

8. $2(x - 2) - x = 12$

9. $-4(x + 2) - 2x = -16$

10. $5(x - 3) + 4x = -6$

11. $2(3x + 2) - x = -6$

12. $5(x - 1) - 4x = 12$

13. $-2(4x + 1) + 2x = 10$

14. $9(3x - 5) = 9$

15. $2(5c - 2) - 2c = 3(2c + 3) + 7$

16. $3(h - 6) = 3(5 - 2h)$

17. $5(x - 2) = 3(x + 4)$

Exercise 3 → ...and some word problems !!

Translate into linear equations and solve:

1. When a number is doubled and the result is increased by 6, the answer is 20. Find the number.
2. The sum of two consecutive integers is 75. Find the integers.
3. The sum of three consecutive even integers is 54. Find the largest of them.
4. When a number is subtracted from 40, the result is 14 more than the original number. Find the number.
5. When 22 is subtracted from a number and the result is doubled, the answer is 6 more than the original number. Find the number.

Solving fractional equations

To solve fractional equations

1. “Get rid of the denominators”
2. “Solve like a normal equation”

Get rid of the denominators

1. Choose the common denominator for the equation.
2. Multiply **EVERY TERM** in the equation by the lowest common denominator.

Examples

1. Solve: $\frac{2x}{5} + 1 = \frac{13}{5}$

The LCM is 5, so I multiply **every term** by 5

$$\frac{2x}{5} + 1 = \frac{13}{5}$$

$$\frac{\overset{1}{\cancel{5}} \cdot 2x}{\underset{1}{\cancel{5}}} + 5 \cdot 1 = \frac{\overset{1}{\cancel{5}} \cdot 13}{\underset{1}{\cancel{5}}}$$

$$2x + 5 = 13$$

$$2x = 13 - 5$$

$$x = 4$$

Remember to multiply **every term** by 5, including the 1.

2. Solve: $\frac{x}{3} - \frac{2x}{5} = \frac{-7}{15}$

The LCM is 15, so I multiply every term by 15

$$\frac{\overset{5}{\cancel{15}} \cdot x}{\cancel{3}} - \frac{\overset{3}{\cancel{15}} \cdot 2x}{\cancel{5}} = \frac{\cancel{15} \cdot (-7)}{\cancel{15}}$$

$$5x - 6x = -7$$

$$-x = -7$$

$$x = 7$$

Solve: $\frac{5}{x+2} = \frac{1}{x}$

The common denominator is $x(x+2)$. Multiply **every term** by $x(x+2)$:

$$\frac{5}{x+2} = \frac{1}{x}$$

$$x(x+2) \frac{5}{x+2} = x(x+2) \frac{1}{x}$$

$$x \cdot \cancel{(x+2)} \frac{5}{\cancel{x+2}} = \cancel{x} \cdot (x+2) \frac{1}{\cancel{x}}$$

($x \neq 0$ and $x+2 \neq 0$)

$$5x = x + 2$$

$$5x - x = 2$$

$$4x = 2$$

$$x = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$x=0.5$$

Since this problem is in the form of a **proportion**, it can also be solved by using "**cross multiply**". (*In a proportion, the product of the means equals the product of the extremes.*)

$$\frac{5}{x+2} = \frac{1}{x}$$

$$5 \cdot x = (x + 2) \cdot 1$$

$$5x = x + 2$$

$$5x - x = 2$$

$$4x = 2$$

$$x = \frac{2}{4} = \frac{1}{2} = 0.5$$

EXERCISES

1. Solve for each variable

a. $\frac{-2x}{5} - 2x + \frac{x}{3} = -x$

g. $\frac{2 \cdot (\chi + 1)}{3} - \frac{\chi}{2} = \frac{\chi + 2 \cdot (\chi + 2)}{6} - \frac{\chi}{3}$

b. $-5x - \frac{4x}{3} + x = -2(x + 3)$

h. $\frac{\alpha + 3}{2} - \frac{2 \cdot (\alpha + 1)}{3} = \alpha - 5$

c. $3 \cdot (x + 4) + \frac{5x}{2} = \frac{3x}{2}$

i. $\frac{2 + x}{3} - \frac{x + 1}{2} = x + \frac{3x + 1}{6}$

d. $\frac{\alpha + 3}{2} = \alpha - 5$

j. $5(x - 3) + 2x = 7x - 4$

e. $\frac{\alpha + 3}{2} - \frac{2 \cdot (\alpha + 1)}{3} = \alpha - 5$

k. $1 - \frac{x}{4} - \frac{2 - x}{3} = \frac{x}{6}$

f. $2 \cdot (3\omega + 4) + 5 \cdot (3\omega - 5) = 3 \cdot (\omega - 7) + 8$

l. $\frac{5}{x+1} = \frac{1}{x}$