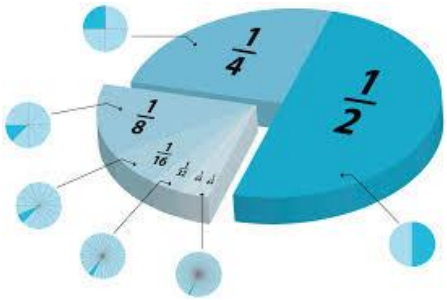
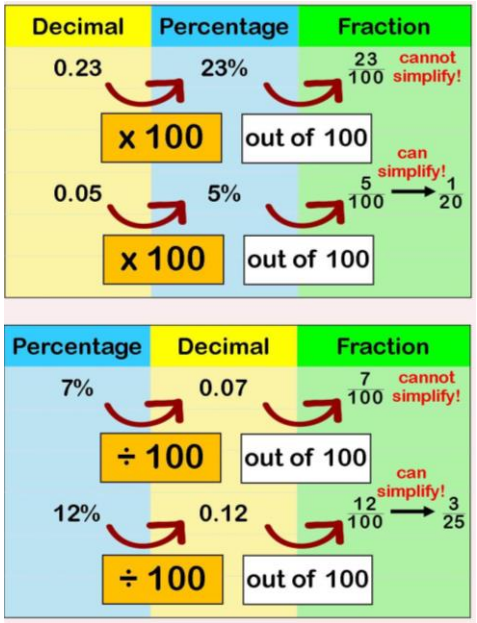
Year 8 – MYP 2

Math

"Manipulating Fractions"





Fractions, Decimals, & Percents

Fraction	Decimal	Percent	Picture
$\frac{1}{10}$	0.1	10%	
$\frac{1}{5}$	0.2	20%	
$\frac{1}{4}$	0.25	25%	
$\frac{1}{3}$	$0.3\overline{3}$	$33.\overline{3}\%$	
$\frac{1}{2}$	0.5	50%	
$\frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}\%$	
$\frac{3}{4}$	0.75	75%	
1	1.00	100%	

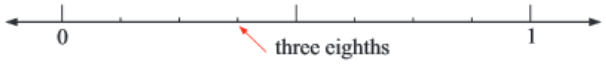
2019-2020

$$fraction = \frac{numerator}{denominator}$$

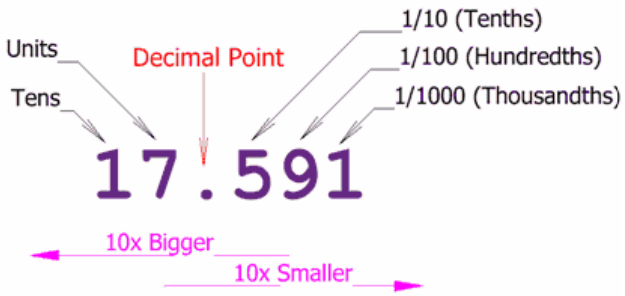
The denominator cannot be equal to zero!!!

Representing Fractions

The fraction three eighths can be represented in a number of different ways:

Words	three eighths
Diagram	as a shaded region *or* as pieces of a pie 
Number line	
Symbol	$\frac{3}{8}$ numerator, bar, denominator

Decimal Numbers



Percentages

Percent (or per cent) means one hundredth. The symbol for percent is %.

Therefore, 1% means 1/100 or one hundredth, and 5% means 5/100 or five hundredths.

The words "per cent" actually mean "per hundred" in Latin.

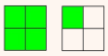
All fractions and decimals can be converted into percentage form by first writing them as fractions with a denominator of 100.

In general, $\frac{a}{100} = a\%$

PROPER AND IMPROPER FRACTIONS - MIXED NUMBERS

A fraction which has numerator less than its denominator is called a **proper fraction**.
For example, 4/5 is a proper fraction and represents 4 equal parts out of 5.

A fraction which has numerator greater than its denominator is called an **improper fraction**.
For example, 5/4 is an improper fraction and represents 1 whole and one part out of 4.



When an improper fraction is written as a whole number and a fraction, it is called a mixed number.
For example, $\frac{5}{4} = 1\frac{1}{4}$

RATIONAL NUMBERS

A **rational number** is a number which can be written in the form $\frac{a}{b}$ where a and b are both integers and $b \neq 0$. We can see that rational numbers are another special type of fractions.

NEGATIVE FRACTIONS

whenever we divided a positive by a negative, or a negative by a positive, the result is a negative. Since the bar of a fraction indicates division, the fraction $\frac{-1}{4}$ means $(-1) \div 4 = -\frac{1}{4}$

It also means $\frac{1}{-4}$ means $1 \div (-4) = -\frac{1}{4}$

So:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

SIMPLIFYING FRACTIONS

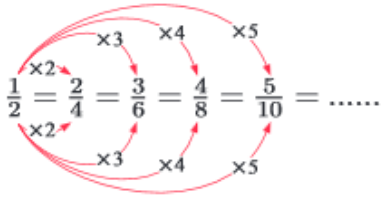
We can simplify a fraction by cancelling common factors in the numerator and denominator. When a fraction is written as a rational number with the smallest possible denominator, we say it is in **lowest terms**.

For example: $\frac{5}{30} = \frac{1\times5}{6\times5} = \frac{1}{6}$

$$\frac{3-7}{2^3+10} = \frac{-4}{8+10} = \frac{-4}{18} = -\frac{2\times2}{9\times2} = -\frac{2}{9}$$

Two fractions are **equal or equivalent** if they can be written in the same lowest terms. We can convert a fraction to an equivalent fraction by multiplying or dividing both the numerator and denominator by the same non-zero number.

For example: $\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$



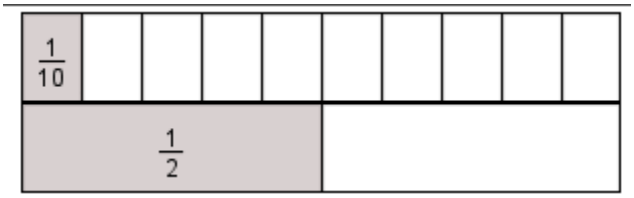
COMPARING FRACTIONS

Sometimes we need to compare two fractions to discover which is larger or smaller. There are three ways to compare fractions:

1. Doing a graphical representation of the fractions
2. using decimals
3. or using the same denominator.

Graphical Representation

We want to compare for example $\frac{1}{10}$ and $\frac{1}{2}$



As you see above, we can represent the fractions graphically and easily compare them.

It's obvious that $\frac{1}{10} < \frac{1}{2}$

Using Decimals

Fractions can be converted into a number (decimal or integer) by dividing the numerator with the denominator.

$\frac{1}{10} = 1 \div 10 = 0.1$

$\frac{1}{2} = 1 \div 2 = 0.5$

It's obvious that $\frac{1}{10} < \frac{1}{2}$

The "Lowest Common Denominator" method.

To compare the fractions, we create equivalent fractions that have a common denominator.

To do that we follow these steps:

1. We find the **LCM** of the denominators.
   Example : For $\frac{3}{10}$ and $\frac{1}{2}$, LCM(10,2)=**10**

2. We put a small hat over each fraction. Inside the hat is the number by which we have to multiply the denominator so as to get the LCM (in this case 10)

   $\frac{\overset{\frown}{\mathbf{1}}}{\frac{3}{10}}$ (The number inside the hat is 1 because 1 x 10=10)

   $\frac{\overset{\frown}{\mathbf{5}}}{\frac{1}{2}}$ (The number inside the hat is 5 because 5 x 2=10)

3. We multiply both the numerator and the denominator of each fraction by the number in the hat

   $\frac{\overset{\frown}{\mathbf{1}}}{\frac{3}{10}} = \frac{3\ x\ \mathbf{1}}{10\ x\ \mathbf{1}} = \frac{3}{10}$

   $\frac{\overset{\frown}{\mathbf{5}}}{\frac{1}{2}} = \frac{1\ x\ \mathbf{5}}{2\ x\ \mathbf{5}} = \frac{5}{10}$

4. Now that the fractions have a common denominator it's easy to compare them. The fraction that has the biggest numerator is the biggest one
   It's obvious that $\frac{3}{10} < \frac{5}{10}$ **so** $\frac{3}{10} < \frac{1}{2}$

Understanding the "Lowest Common Denominator Method" is very important since we will be using this method whenever we have to add or subtract fractions.

Fun time : On line comparing fractions game 
http://www.arcademics.com/games/dirt-bike-comparing-fractions/dirt-bike-comparing-fractions.html

Exercises

1. Write the following fractions as mixed numbers and the opposite:

4/3	
	$1\frac{1}{4}$
	$4\frac{2}{5}$
7/5	
9/8	
11/4	
	$6\frac{3}{5}$
	$5\frac{1}{2}$

2. Suppose we have two dice. We roll one to give the numerator of a fraction and the other to give the denominator. Find:

a) the smallest fraction it is possible to roll
b) the largest proper fraction it is possible to roll
c) the largest improper fraction which is not a whole number that it is possible to roll
d) the number of different fractions it is possible to roll.

$$\frac{numerator}{denominator}$$

3. Express with denominator 12:

i. $\frac{2}{3}$

ii. $\frac{5}{6}$

iii. $\frac{15}{45}$

iv. $\frac{6}{18}$

v. $\frac{18}{42}$

vi. $\frac{240}{280}$

4. Simplify as far as possible:

i. $\frac{21}{3}$

ii. $\frac{-5}{60}$

iii. $\frac{15}{-453}$

iv. $-\frac{16}{18}$

v. $\frac{8}{36}$

vi. $\frac{-24}{280}$

vii. $\frac{2}{-54}$

viii. $\frac{-4}{60}$

ix. $\frac{-8}{72}$

x. $-\frac{18}{3}$

5. Plot each set of fractions on a number line:

i. $\frac{2}{-3}$, $\frac{5}{6}$, $\frac{-5}{30}$, $1\frac{1}{3}$

ii. $-\frac{4}{3}$, $\frac{5}{60}$, $\frac{-2}{12}$, $\frac{1}{4}$

6. Simplify:

a. $\frac{3^2}{4-7}$

b. $\frac{3^2-2\times 5}{8-3^2}$

c. $\frac{2^2-5}{-16\div 2^2}$

d. $\frac{11-3}{32-2^4}$

e. $\frac{8-2\times 5}{4\times 3\ -4}$

f. $\frac{6-(-12)}{4-7}$

g. $\frac{-5\times(-6)-(-15)}{2^2-3^2}$

h. $\frac{3^3+(-3)}{-16\div 2}$

Comparing Fractions Exercises

7. Compare the following fractions and mixed numbers. Write between them the correct symbol "<" , ">" , "=" .

a. $\frac{1}{5}$ $\frac{1}{4}$

b. $\frac{1}{2}$ $\frac{4}{5}$

c. $\frac{1}{5}$ $\frac{2}{10}$

d. $\frac{1}{2}$ $\frac{3}{4}$

e. $\frac{11}{12}$ $\frac{11}{13}$

f. $\frac{2}{3}$ $\frac{5}{6}$

g. $\frac{1}{5}$ $\frac{3}{5}$

h. $\frac{123}{124}$ $\frac{5}{4}$

i. $1\frac{1}{5}$ $2\frac{1}{5}$

j. $\frac{6}{5}$ $1\frac{1}{5}$

k. $\frac{11}{5}$ $2\frac{2}{10}$

l. $3\frac{1}{6}$ $3\frac{3}{18}$

8. Write the following fractions in ascending order (from smallest to biggest)

a. $-\frac{1}{4}$ $\frac{4}{12}$ $\frac{1}{2}$ $\frac{4}{4}$ $-\frac{6}{4}$ $1\frac{3}{4}$

b. $\frac{1}{-3}$ $\frac{2}{5}$ $\frac{2}{-6}$ $\frac{5}{5}$ $\frac{6}{5}$ $\frac{-8}{10}$

c. $\frac{2}{3}$ $\frac{-5}{12}$ $\frac{6}{-12}$ $\frac{1}{6}$ $\frac{-12}{12}$ $1\frac{3}{6}$

d. $\frac{3}{4}$ $\frac{-15}{6}$ $\frac{1}{8}$ $-1\frac{2}{4}$ $\frac{-7}{24}$ $1\frac{3}{4}$