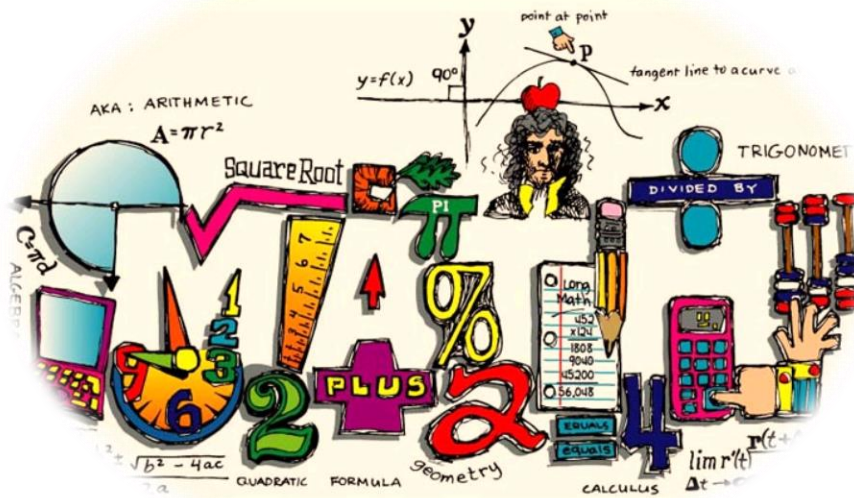


Year 8 - MYP 2

Math

"Properties of numbers"



2019-2020

Even and Odd number

A natural number is **even** if it has 2 as a factor and thus is divisible by 2.

A natural number is **odd** if it is not divisible by 2.

Divisibility rules for natural numbers

Number	Divisibility test
2	If the last digit is 0 or even, then the original number is divisible by 2
3	If the sum of the digits is divisible by 3, then the original number is divisible by 3
4	If the number formed by the last two digits is divisible by 4, then the original number is divisible by 4
5	If the last digit is 0 or 5, then the original number is divisible by 5
6	If the number is divisible by both 2 and 3 then is divisible by 6.
8	If the number formed by the last three digits is divisible by 8, then the original number is divisible by 8
9	If the sum of the digits is divisible by 9, then the original number is divisible by 9
10	If the last digit is 0, then the original number is divisible by 10

Factors

The *factors* of a number are all the numbers which go into the number exactly, with no remainder. Remember that **1 and the number itself** are always factors of any number, and these two factors are called *trivial factors* because they are obvious.



Example: The factors of 6 are 1, 2, 3 and 6. (6 can be divided by 1,2,3 and 6)

Common Factors

If we have a list of numbers a **common factor** is any number which is a factor of every number in the list.



Example: 4 and 7 are both common factors of 28, 56 and 84.

Highest Common Factor

The Highest Common Factor (HCF) is the highest number that is a factor of every number in the list

Tip : Remember that the HCF is a factor of every number in the list, so it must be less than or equal to the smallest number in the list



Example: the HCF of 12, 16, and 20 is 4.

Prime Numbers

A *prime number* has no factors except 1 and itself. A *prime number must have two factors, 1 and itself, so 1 is not a prime number.*



Example: 2, 3, 5 and 7 are all prime numbers.

The first 8 prime numbers are :

2 , 3 , 5 , 7 , 11 , 13 , 17 , 19



!! It is helpful to be able to recognise them. !!



Composite Numbers

A number which has factors other than 1 and itself is called a composite number



Example: 6 is a composite number because the factors of 6 are 1, 2, 3, 6. (6 can be divided by 1, 2, 3 and 6).

Prime Factors

The prime factors of a composite number are all the factors of the number which are prime numbers



Example: the factors of 18 are 1, 2, 3, 6, 9 and 18 but the prime factors are 2 and 3 only (since 2 and 3 are primes).



Zero and one are considered neither prime nor composite numbers- they're in a class by themselves!

Multiples

A multiple of a number is any number that it goes into it. The multiples of a number are therefore the numbers that make up its "times table".



Example: 6, 12, 18, 24, 30 and 36 are all multiples of 6

Common Multiples

A common multiple of a list of numbers, is a number which is a multiple of all of them.



Example: 12 and 24 are common multiples of 2, 3, 4 and 6.

Lowest Common Multiple (LCM)

The Lowest Common Multiple (LCM) of a list of numbers is the lowest number that is a multiple of all of them.



Example: although 24 and 36 are multiples of 2, 3 and 4 the LCM is 12

Tip : the LCM is a multiple of every number in the list so it must be greater than or equal to the largest number in it.

Reminder: Prime Numbers: 2, 3, 5, 7, 11, 13, 17



Example 1

This is how we can write a number as a product of prime factors.

E.g. Find the prime factors of 120

2	120
	60


2	120
2	60
	30

2	120
2	60
2	30
3	15
5	5
	1

Start by finding the smallest prime number that will divide into the number. Divide it!

Now find the smallest prime number that will divide into the answer. Divide it!

Keep dividing the answers by the smallest prime number. Stop when you reach 1.

So the Prime factors of 120 are 2, 3, 5
 And $120 = 2 \times 2 \times 2 \times 3 \times 5$
 In index notation: $120 = 2^3 \times 3 \times 5$ 

Exercise 1

Find the prime factors of these numbers.

- a. 8
- b. 10
- c. 20
- d. 30
- e. 54

Example 2

This is how we can find the HCF (Highest Common Factor) of a number.

E.g. Find the HCF of 24, 36, 12

Method 1

Steps:

1. We find the factors of each number

The factors of 24 are: 2, 4, 6, **12**, 24

The factors of 36 are: 2, 4, 6, 9, **12**, 18

The factors of 12 are: 2, 4, 6, **12**

2. We see that **common factors** are **2, 4, 6** and **12**
3. The **highest common factor (HCF)** is **12**

Method 2

1. Let's write the numbers as products of prime numbers. (As we learned on the previous page)

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$12 = 2 \times 2 \times 3$$

2. If we combine the common ones (the numbers that are the same in all three lists) they will give us the highest common factor of 12, 24 and 36:
3. The **highest common factor (HCF)** of **12, 24, 36** is **$2 \times 2 \times 3$**
= 12

Exercise 2

Find the HCF of 16 and 56

Example 3

This is how we can find the LCM (Least Common Multiple) of a number.

E.g. Find the LCM of 4, 6, 8

Method 1

Steps:

1. We find the multiples of each number

The multiples of 8 are: 8, 16, **24**, 32, 40, 48

The factors of 6 are: 6, 12, 18, **24**, 30, 36, 42, 48

The factors of 4 are: 4, 8, 12, 16, 20, **24**, 28, 32, 36, 40, 44, 48

2. We see that the **least common multiple** is **24** (There are more common multiples, but this is the smallest one)

Method 2

Look at the following example:

4	8	12	2
2	4	6	2
1	2	3	2
1	1	3	3
1	1	1	

The **least common multiple** is $2 \times 2 \times 2 \times 3 = 24$

(We will describe this method more thoroughly in class)

Exercise 3

Find the LCM of 8, 12, 20

Exercise 4

Which of the following numbers are divisible by 3:

- | | | | |
|---------|-----------|--------|-----------|
| a. 87 | b. 153 | c. 145 | d. 891 |
| e. 1002 | f. 877992 | g. 111 | h. 111222 |

Exercise 5

Determine whether the following numbers are divisible by 2, 3, 4, 5, 6, 9, 10

125 540 789 12321 6780 675

Exercise 6

Fill in the blanks:

- (i) Even numbers are the multiples of _____.
- (ii) 24 is an even number but 29 is an _____ number.
- (iii) _____ is neither prime nor composite number.
- (iv) _____ is the only even prime number.
- (v) The smallest prime number is _____.
- (vi) The smallest odd composite number is _____.
- (vii) There are _____ prime numbers between 1 and 15.

Exercise 7

- a) List all the prime numbers less than 30
- b) List all the factors of the following numbers: 5, 26
- c) Show the following number as a product of its prime factors
 - i. 18
 - ii. 21
 - iii. 57

- d) Show the following number as a product of its prime factors in index form: 40
- e) Find the Highest Common Factor of the following pair of numbers
15 and 6
16 and 12
- f) Find the Lowest Common Multiple of the following pair of numbers
9 and 15
36 and 38

Exercise 8

Test whether the numbers in red colour are the multiples of the numbers in blue colour. Put (✓) if yes, and (X) if no in the blank space.

- (i) 40 3 _____
- (ii) 75 5 _____
- (iii) 49 6 _____
- (iv) 99 9 _____
- (v) 48 7 _____
- (vi) 50 2 _____

Exercise 9

Tamara has two pieces of ribbon of lengths 18 cm and 24 cm respectively. She wants to cut both pieces into smaller pieces of equal length that are as long as possible. What would be the length of each smaller piece?